MATHEMATICIANS' WRITING

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Computer systems that support collaboration among students are not easily adopted in mathematics education. A possible reason for this is that the computer is rarely the students' preferred writing tool in mathematics. This paper investigates the writing processes of two research mathematicians, and tries to understand what parts of the mathematical writing process it is relevant and possible to support with a computer, for research mathematicians as well as students of mathematics.

INTRODUCTION

In recent years many educational institutions have introduced computer systems to support collaboration among students. The use of such systems is, in the literature, referred to as Computer Supported Collaborative Learning or CSCL (see Koschmann, 1996). These systems seem to have had some success in many areas, but are typically not easily adopted in mathematics (see Guzdial et al., 2002). This somewhat contrasts with the results from a large interview-based study by Leone Burton (1999) on how research mathematicians come to know mathematics, revealing that the discipline is moving towards a more collaborative nature (see Burton, 1999 p. 137). The reason for the lack of success in introducing computer systems to support collaboration in mathematics education could be that mathematical notation is difficult or impossible to write in many computer systems. This problem was approached in Guzdial et al. (2002) by constructing a formulae editor for a very successful CSCL system (the CoWeb). The mathematics students did not adopt this improved system, which suggests that either the students did not want to collaborate or that the possibility of writing formulas was somehow insufficient to facilitate collaboration through such a system (or a combination of the two).

Writing on a computer is often a prerequisite for collaborating through a computer system, and an increasing amount of students' writing, at most levels and in most topics, is done using a computer. But in mathematics this is typically not the case, because the many graphical elements and complicated notation makes it easier to use pen and paper than the computer. Thus the introduction of computer supported collaboration in mathematics education is intimately connected to the development of writing tools that support the mathematical writing process.

This paper investigates the mathematical writing process through the cases of two research mathematicians. There are several reasons to look at research mathematicians' writing processes in order to understand why it is so hard to use computer systems for collaboration in mathematics education. First of all it seems reasonable to search for inspiration for students' collaborative writing processes among research mathematicians, since they according to Burton (1999) collaborate more and more. Another advantage of looking at research mathematicians is that, compared to others, they use computers for writing mathematically to a much larger extent.

Burton & Morgan (2000) have, in connection with the study described in Burton (1999), investigated the language used in mathematical research papers. Their focus is on the natural language used in finished research papers and their goal is partly to describe what type of writing is acceptable in the mathematical community (see Burton & Morgan, 2000 p.432). Where Burton & Morgan (2000) describe the natural language used in the finished paper, this study is concerned with the full range of representations and tools used by the mathematician throughout the entire writing process. And where Burton (1999) is an epistemological study, this study is more pragmatic in the sense that it describes existing practices of mathematicians and because it is motivated by the practical problem of why it is so hard to make students use a CSCL system in mathematics.

What this study does share with Burton (1999) and Burton & Morgan (2000) is an underlying assumption that a closer look at the practice of research mathematicians can reveal information of potential importance for mathematics education.

QUESTIONS

The main question of my research is: Why is it so difficult for mathematics students to collaborate through a computer system? And, can new technology or a different use of existing technology make it easier for students to collaborate in this way? These questions will not be answered completely here. But I will try to get closer to an answer by describing the writing processes of research mathematicians from an early idea to a finished paper. In particular, I will compare the purposes writing serves for the mathematician (e.g. to save information, work out a calculation, communicate to a collaborator etc.) with the types of representations he/she uses and the media (computer or pen and paper) he/she chooses.

METHODOLOGY

In this research report I present two cases to illustrate two somewhat different approaches to writing in mathematical research. Both cases are part of a larger interview-based study on the writing processes of research mathematicians, and the purpose of this report is partly to develop a framework for this larger study. I will describe the methodology of that larger study here. I have met all the participants in their offices where I interviewed them. The length of the interviews ranged from approximately half an hour to a little more than one hour, but most of the interviews lasted about 45 minutes. The interviews were organized as conversations around two main questions concerning (1) communication during collaborative research projects and (2) the use of writing in the personal research process. During the conversations the respondents sometimes referred to papers on their desks or in their archives. If a sheet of paper was especially interesting, for example because it contained a form of representation that was new to me, or because we had had an interesting conversation about it, I would ask for a photocopy of that sheet.

The interviews were taped and transcribed. In the analysis of the interviews I brought in samples of working papers where relevant.

RESULTS

In the following I account for the writing processes of two mathematicians "Peter" and "John". I focus on the work they do when they are alone, without face-to-face contact with collaborators, where most communication is done using a computer.

Stepwise Writing: The Case of Peter

Peter uses three different media for writing mathematics (two paper-based and one electronic) and he clearly classifies his work according to the medium in use. The media are blank scratch paper (the backside of old printouts) for handwriting, a lined pad also for handwriting and his computer with an e-mail program and LaTeX [1].

I followed Peter through a short writing process. When I first came to see him he explained that he had recently been thinking about a problem that he was working on together with a collaborator. He had the main idea worked out and had just begun the process of writing it out in detail. I received a photocopy of his detailed draft three days later and after a week I was cc'ed with a first edition of an article on the subject, that he e-mailed to his collaborator. Soon after that I interviewed him again.

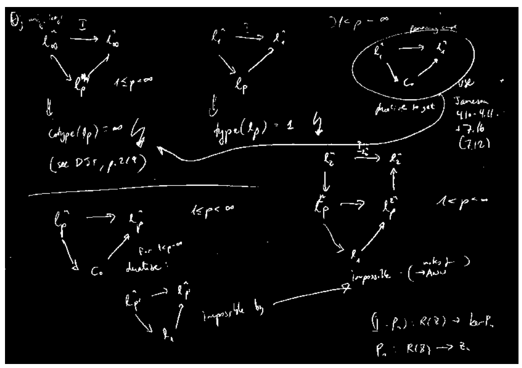

Figure 1: Scratch paper

Peter's office contained a whiteboard next to a large bookshelf and across from that a small desk with Peter's computer next to the main desk[2]. When I first visited Peter, on the main desk, there was a pile of scratch paper (backside of old printouts), a folder, a pad of lined paper and two books. One sheet of scratch paper was placed in the middle of the desk and filled with scribbles (see figure 1). The folder was filled with sheets from the lined pad, and on the lined pad there was handwritten mathematical text (like the sheet to the left in figure 2).

Peter uses the scratch paper for personal scribbles, and he explains that these papers only make sense while he is in the process of working on a problem, and that most of them are

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[1] LaTeX is a typesetting system widely used in the mathematical community. The author will write a source code in a text editor and process it to a print file. So the author will write:

`$\int_a^b  f(t)dt=F(b)-F(a)$` in a text editor, to typeset: $\int_a^b f(t)dt = F(b) - F(a)$ in the print file.

[2] The fact that the computer was on a separate desk is common among research mathematicians.

thrown away almost immediately. The content of the scratch paper has no obvious linear structure (see Morgan, 1997 p. 89); it consists of scribbles scattered around the paper.

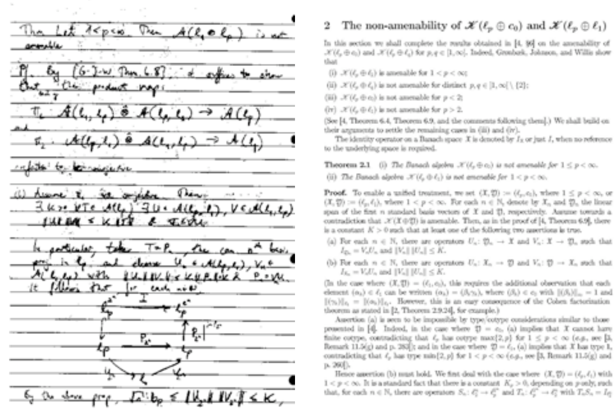

Figure 2: Lined pad and LaTeX version

Peter explains, “If I believe something [from the scratch papers] seems to be working out right then I will take the lined pad and try to write down as many details as possible.” Again, when that is done the scratch papers are thrown away but the lined paper with all the details are kept carefully in a system of folders. Peter explains that the purpose of writing it out in detail is twofold: to make the work accessible later, and to check in detail if his ideas are correct.

When Peter thinks he has enough work for an article, or a part of one, he will write a LaTeX version of his work. He describes that version as “much shorter, to fit an article”. The computer version is sent to collaborators for comments and proofreading, but the main reason for making the LaTeX version is to produce a paper.

Peter believes that the notes on lined paper in his folders contain more information than the finished paper. He keeps the notes partly to be able to go back and investigate ideas he never published, and also so he can always go back and check how he got to a specific result. Peter imagines that this would be practical if confronted with questions of how he got to a specific result, both to be able to defend his results and more easily acknowledge if he made a mistake.

Peter communicates with collaborators mainly by e-mail. He explains that the content is often very close to the content of the notes on lined paper that he keeps in his folders. To be able to express mathematics in an e-mail he will often use LaTeX code[3] in the e-mail. This gives rise to some extra work with moving the content to another medium. To avoid this work, Peter has tried to fax the notes from the folder, but has had difficulties with

[3] In e-mails a special use of the LaTeX commands has developed. Since e-mail does not support mathematical symbols it is common to use the LaTeX command whenever a symbol is needed, this style, sometimes denoted “pseudo LaTeX”, seems to be fully accepted in e-mail communication among mathematicians.

this. Peter explains that if, when writing, he had to think about how it could be faxed (i.e. choose a dark pen, avoid using the margin, think about a potential reader etc.), this would disturb his thinking and he would be less able to concentrate on checking the mathematical details of his idea.

Successive Writing: The Case of John

In the early stages of his writing, John uses paper and pen. He describes the "first treatment/phase one" work as being inaccessible to others and not for archiving, at least not without some explanations[4]. What John will do next then depends on how things proceed. If he gets to a publishable result he will very soon write an early version in LaTeX, and then start to work with pen and paper on a printout version of that, successively adding to and correcting the LaTeX draft, which therefore evolves dramatically over time. John explains that he deliberately does not try to get it right from the beginning. On the one hand, John writes the LaTeX draft to save his work and start the production of an article. On the other hand, the contents develop over time; the creative work is not over when the first draft is written in LaTeX. John argues that this way of working suits him better than trying to have it all figured out from the beginning.

If John does not arrive at something publishable he will sum up his work and save it in a large metal drawer. This is done with pen and paper, the same type of paper as he uses in the first phase. The summary consists of things like "I have been working on x using the approach y; I got stuck there." Apart from these overviews the summary consists of annotations and re-writings of some of the early stages of his research. The purpose of this summary phase is to make a saveable version of his early scribbling.

ANALYSIS

Functions of Writing Mathematically

The writing processes of Peter and John have led me to consider the following five different functions of writing in mathematics explained below. By distinguishing between these five functions I do not claim that they are mutually disjointed or that they completely cover what mathematical writing is. Rather, I try to give a framework for further analysis of the mathematical writing process.

(1) *Heuristic treatment* consists of getting and trying out ideas and seeing connections. (2) *Control treatment* is a deeper investigation of the heuristic ideas. It can have the form of pure control of a proposition or be a more open-ended investigation (e.g. a large calculation to find x). It is characterized by precision. (3) *Information storage* is to save information for accessing later. (4) *Communication* with collaborators and (5) *Production* of an article.

The first three functions are inspired by Duval (2000), who distinguishes two significant ways in which writing expands our cognitive abilities: To help save information and to support ongoing mental processes. Both heuristic treatment and control treatment support

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[4] He even explains how he sometimes tries to take up a project that has been sitting for a month and finds that his notes are unusable.

ongoing mental processes, and I believe the cases show that it is reasonable to consider them separately.

The Cases

With the framework from the previous section it is possible to describe the writing processes of Peter and John in some detail. In the following figures I have tried to map a timeline of how the functions of heuristic treatment, control treatment, and production come into play, and in which medium this occurs. Each box describes a specific medium used for a certain function at a certain stage, and is annotated with information regarding information saving and communication during this stage of work.

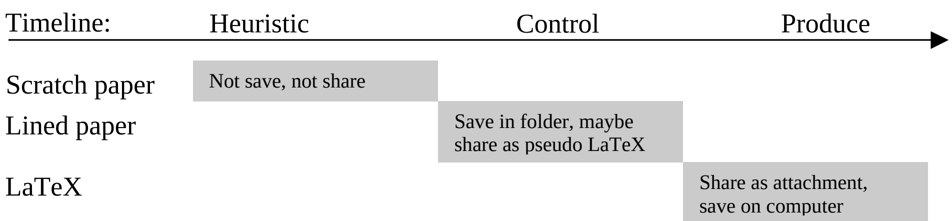

Figure 3: Diagram of Peter's writing process.

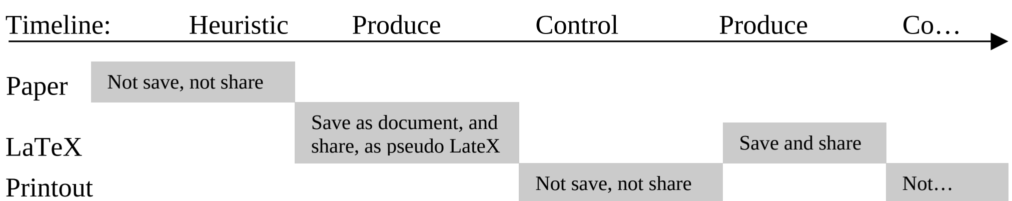

Figure 4: Diagram of John's writing process in the case where he arrives at something he decides could be an article. Note that in the last phases of his work he will switch back and forth between working with LaTeX and with a pen on a printout of his document.

Both Peter and John use pen and paper for heuristic treatment. But whereas Peter uses one type of paper for heuristic treatment and another type for control treatment, John just uses paper. John often combines pen and paper with LaTeX drafts in the control treatment phase. To save information Peter uses papers in folders and John uses either an evolving LaTeX draft or pieces of paper from heuristic or control treatment annotated in order to be saveable. They both communicate with collaborators by e-mail, sometimes attaching LaTeX files. To produce articles they both use LaTeX. But where Peter basically sits down and transcribes a paper draft to the computer in order to write an article, John's work will evolve from control treatment and personal saved work towards an article as a LaTeX file.

Both Peter's and John's working papers develop from containing mainly symbolic language in the heuristic phase towards the extended use of natural language in the control phase. But where the samples from Peter have a clear movement from nonlinear (see Morgan, 1998 p.89) writing in the heuristic phase towards linearity in the control and production phase, John's working papers are linear throughout the writing process. For Peter, the control treatment is intimately connected to saving information and communication with collaborators, whereas the heuristic treatment does not seem to be connected to those functions. The production of an article is connected to communication with collaborators because Peter sends his drafts to his collaborators for comment, but production does not seem to be connected to either heuristic or control treatment. For John, there is a strong connection between the production of an article, the saving of information and control treatment because he saves his work in an evolving LaTeX document and checks details by working with a printout of his LaTeX document.

EDUCATIONAL PERSPECTIVES

Pen and paper play a central role in the heuristic phase of the two mathematicians' work; neither uses a computer in that phase. This suggests that the existing computer systems do not support heuristic mathematical writing among researchers. The mathematical activities of students, as well as their use of computers, are of course different from those of researchers, but one can at least question whether students will benefit from using the existing technology for heuristic writing when researchers do not. Since none of the mathematicians use the heuristic phase directly for saving information[5] or for communication, it can be argued that computer support for heuristic mathematical writing is not essential for the success of computer supported collaborative learning environments.

The control treatment seems to be highly connected with communication and the saving of information, and both of the researchers are able to use computers to support that function. Therefore the development of better tools to support writing that serves a control treatment function seems to be relevant for the success of CSCL in mathematics.

Even though the cases show some similarities in the mathematical writing process, they definitely also show diversity. Students are also different and have different writing processes. I find it especially important to note that this framework does not suggest one canonical "best sequence of phases" in mathematical research. Rather than using technology to encourage or force students to approach problems in one specific way, I would suggest that the students, while working with mathematics, should have access to the best possible support for their writing at all times, and learn (implicitly or explicitly) to choose the kind of support that best suits the task at hand, acknowledging that this support will be different for different persons.

It goes without saying that as long as pen and paper are the primary support for one or more of the functions in mathematical writing (as seems to be the case at the moment) the students should have access to this medium at all times. Doing mathematics in computer labs without sufficient desk space is a problem when some essential work in mathematics is best done with pen and paper.

I believe that an increased awareness of the nature of mathematical writing processes can be relevant to mathematics education in general. The fact that at least some research mathematicians do not start producing their work before they have gone through a heuristic and perhaps also a control phase, could perhaps be relevant to the problem of

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[5] John actually uses some of his sheets from the heuristic phase in his archive, but these sheets go through major revision (annotation and re-writing) in order to be saveable.

how to introduce writing in mathematics education. For example, can the well-known problem of students writing too little text to answer word problems be described in the framework suggested here as students delivering their work too early rather than it being incorrect. So rather than encouraging a student to write more while working on a solution to the problem, thus giving the student the impression that he/she solved the problem incorrectly, the teacher might make the student aware that his/her work is proceeding well but needs to be written out in more detail in order to make sure that the results are correct and can be read by others. A different but related problem is the use of pre-made worksheets for students to fill out with solutions to problems. The sheets are made to help the students, but can end up disturbing their creative process by forcing them to think about the finished product (e.g. readability) too early in the process.

It is not entirely clear how an analysis of the writing and working processes of research mathematicians is relevant for mathematics education. On the one hand, many typical activities in the mathematics classroom greatly differ from mathematical research. On the other hand, I think that an awareness of the mathematicians' strategies and choice of medium in different stages of the writing process can give a new perspective on students' problems in mathematics. Moreover, I believe the problems that research mathematicians have in using computers for writing mathematically are potentially relevant for the introduction of computers in mathematics education.

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